15. A line, $L_1$, passes through the point $P(2, 4, 1)$ and is parallel to

$$u_1 = i + 2j - k$$

and a second line, $L_2$, passes through $Q(-5, 2, 5)$ and is parallel to

$$u_2 = -4i + 4j + k.$$ 

(a) Write down the vector equations for $L_1$ and $L_2$.

(b) Show that the lines $L_1$ and $L_2$ intersect and find the point of intersection.

(c) Determine the equation of the plane containing $L_1$ and $L_2$.

Answers

(a)

$$u_1 = i + 2j - k \quad \text{direction vector} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$u_2 = -4i + 4j + k \quad \text{direction vector} \begin{pmatrix} -4 \\ 4 \\ 1 \end{pmatrix}$$

$$(1) \quad v_1 = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -5 \\ 2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 4 \\ 1 \end{pmatrix}$$

(b) Find $z_1$ & $z_2$

Since $z_1 = z_2$ the same the lines intersect as $(-1, -2, 4)$

(c) $6x + 3y + 12z = 36 \quad \text{or} \quad 2x + y + 4z = 12$