2013 Q16

16. In an environment without enough resources to support a population greater than 1000, the population \( P(t) \) at time \( t \) is governed by Verhurst’s law

\[
\frac{dP}{dt} = P(1000 - P).
\]

Show that

\[
\ln \frac{P}{1000 - P} = 1000t + C \quad \text{for some constant } C.
\]

Hence show that

\[
P(t) = \frac{1000K}{K + e^{-1000t}} \quad \text{for some constant } K.
\]

Given that \( P(0) = 200 \), determine at what time \( t \), \( P(t) = 900 \).

Answers

Proof – First two parts of question

\[
t = \frac{1}{1000} \ln 36 = 0.003584 \ (4 \text{ s.f.})
\]