2010 Q12

12. Prove by contradiction that if $x$ is an irrational number, then $2 + x$ is irrational.

Answer

Assume $2 + x$ is rational
and let $2 + x = \frac{p}{q}$ where $p$, $q$ are integers.

So

$$x = \frac{p}{q} - 2$$

$$= \frac{p - 2q}{q}$$

Since $p - 2q$ and $q$ are integers, it follows that $x$ is rational. This is a contradiction.