12. The first two terms of a geometric sequence are \( a_1 = p \) and \( a_2 = p^2 \). Obtain expressions for \( S_n \) and \( S_{2n} \) in terms of \( p \), where \( S_k = \sum_{j=1}^{k} a_j \).

Given that \( S_{2n} = 65S_n \) show that \( p^n = 64 \).

Given also that \( a_3 = 2p \) and that \( p > 0 \), obtain the exact value of \( p \) and hence the value of \( n \).

Answers

\[
S_n = \frac{p(p^n - 1)}{p - 1} \quad \quad S_{2n} = \frac{p(p^{2n} - 1)}{p - 1}
\]

\( p = \sqrt{2} \)

\( n = 12 \)