2007 Q16

16. 

(a) The diagram shows part of the graph of \( f(x) = \tan^{-1} 2x \) and its asymptotes. State the equations of these asymptotes.

(b) Use integration by parts to find the area between \( f(x) \), the \( x \)-axis and the lines \( x = 0 \), \( x = \frac{1}{2} \).

(c) Sketch the graph of \( y = |f(x)| \) and calculate the area between this graph, the \( x \)-axis and the lines \( x = -\frac{1}{2} \), \( x = \frac{1}{2} \).

Answers

(a) Horizontal asymptote at \( y = \pm \frac{\pi}{2} \)

(b) \[ \text{Area} = \frac{\pi}{8} - \frac{\ln 2}{4} \]

(c) 

\[ \text{Area} = \frac{\pi}{4} - \frac{\ln 2}{2} \]