2007 Q12

12. Prove by induction that for $a > 0$,

$$(1 + a)^n \geq 1 + na$$

for all positive integers $n$.

Answer

Consider $n = 1$, LHS = $(1 + a)$, RHS = $1 + a$ so true for $n = 1$.
Assume that $(1 + a)^k \geq 1 + ka$ and consider $(1 + a)^{k+1}$.

$$(1 + a)^{k+1} = (1 + a)(1 + a)^k$$

$$(1 + a)(1 + ka)$$

$= 1 + a + ka + ka^2$$

$= 1 + (k + 1)a + ka^2$$

$> 1 + (k + 1)a$ since $ka^2 > 0$

as required. So since true for $n = 1$, by mathematical induction statement is true for all $n \geq 1$. 