Vectors
AH Maths Exam Questions

Source: 2019 Specimen P2 Q13 AH Maths

| (1) | A line, $L$, has equation $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z}{-1}$. |
|     | (a) Find the Cartesian equation of the plane, perpendicular to the line $L$, which passes through the point P(1,1,0). |
|     | (b) Find the shortest distance from P to $L$ and explain why this is the shortest distance. |

Source: 2019 Q15 AH Maths

| (2) | The equations of two planes are given below. |
|     | $\pi_1$: $2x - 3y - z = 9$ |
|     | $\pi_2$: $x + y - 3z = 2$ |
|     | (a) Verify that the line of intersection, $L_1$, of these two planes has parametric equations $x = 2\lambda + 3$, $y = \lambda - 1$, $z = \lambda$. |
|     | (b) Let $\pi_3$ be the plane with equation $-2x + 4y + 3z = 4$. Calculate the acute angle between the line $L_1$ and the plane $\pi_3$. |
|     | (c) $L_2$ is the line perpendicular to $\pi_3$ passing through P(1, 3, -2). Determine whether or not $L_1$ and $L_2$ intersect. |
Planes $\pi_1$, $\pi_2$ and $\pi_3$ have equations:

- $\pi_1$: $x - 2y + z = -4$
- $\pi_2$: $3x - 5y - 2z = 1$
- $\pi_3$: $-7x + 11y + az = -11$

where $a \in \mathbb{R}$.

(a) Use Gaussian elimination to find the value of $a$ such that the intersection of the planes $\pi_1$, $\pi_2$ and $\pi_3$ is a line.

(b) Find the equation of the line of intersection of the planes when $a$ takes this value.

The plane $\pi_4$ has equation $-9x + 15y + 6z = 20$.

(c) Find the acute angle between $\pi_1$ and $\pi_4$.

(d) Describe the geometrical relationship between $\pi_2$ and $\pi_4$.

Justify your answer.

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(a) A beam of light passes through the points $B(7, 8, 1)$ and $T(-3, -22, 6)$.

Obtain parametric equations of the line representing the beam of light.

(b) A sheet of metal is represented by a plane containing the points $P(2, 1, 9)$, $Q(1, 2, 7)$ and $R(-3, 7, 1)$.

Find the Cartesian equation of the plane.

(c) The beam of light passes through a hole in the metal at point $H$.

Find the coordinates of $H$. 
### Source: 2016 Q14 AH Maths

| (5) | Two lines $L_1$ and $L_2$ are given by the equations:  

$$L_1: \quad x = 4 + 3\lambda, \quad y = 2 + 4\lambda, \quad z = -7\lambda$$  

$$L_2: \quad \frac{x-3}{-2} = \frac{y-8}{1} = \frac{z+1}{3}$$  

(a) Show that the lines $L_1$ and $L_2$ intersect and find the point of intersection  

(b) Calculate the obtuse angle between the lines $L_1$ and $L_2$. |

### Source: 2015 Q15 AH Maths

| (6) | A line, $L_1$, passes through the point P(2, 4, 1) and is parallel to  

$$\mathbf{u}_1 = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$$  

and a second line, $L_2$, passes through Q(−5, 2, 5) and is parallel to  

$$\mathbf{u}_2 = -4\mathbf{i} + 4\mathbf{j} + \mathbf{k}.$$  

(a) Write down the vector equations for $L_1$ and $L_2$.  

(b) Show that the lines $L_1$ and $L_2$ intersect and find the point of intersection.  

(c) Determine the equation of the plane containing $L_1$ and $L_2$. |
### 2014 Q5 AH Maths

| (7) | Three vectors $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ are given by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ where $\mathbf{u} = 5\mathbf{i} + 13\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, $\mathbf{w} = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$. Calculate $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$. Interpret your result geometrically. |

### 2013 Q15 AH Maths

| (8) | (a) Find an equation of the plane $\pi_1$, through the points $A(0, -1, 3)$, $B(1, 0, 3)$ and $C(0, 0, 5)$.

(b) $\pi_2$ is the plane through $A$ with normal in the direction $-\mathbf{j} + \mathbf{k}$. Find an equation of the plane $\pi_2$.

(c) Determine the acute angle between planes $\pi_1$ and $\pi_2$. |

### 2012 Q5 AH Maths

| (9) | Obtain an equation for the plane passing through the points $P(-2, 1, -1)$, $Q(1, 2, 3)$ and $R(3, 0, 1)$. |
The lines $L_1$ and $L_2$ are given by the equations

$$\frac{x - 1}{k} = \frac{y}{-1} = \frac{z + 3}{1} \quad \text{and} \quad \frac{x - 4}{1} = \frac{y + 3}{1} = \frac{z + 3}{2},$$

respectively.

Find:

(a) the value of $k$ for which $L_1$ and $L_2$ intersect and the point of intersection;

(b) the acute angle between $L_1$ and $L_2$.

Given $\mathbf{u} = -2\mathbf{i} + 5\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{w} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$.

Calculate $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

(a) Use Gaussian elimination to solve the following system of equations

$$x + y - z = 6$$
$$2x - 3y + 2z = 2$$
$$-5x + 2y - 4z = 1.$$

(b) Show that the line of intersection, $L$, of the planes $x + y - z = 6$ and $2x - 3y + 2z = 2$ has parametric equations

$$x = \lambda$$
$$y = 4\lambda - 14$$
$$z = 5\lambda - 20.$$

(c) Find the acute angle between line $L$ and the plane $-5x + 2y - 4z = 1$. 


### Source: 2008 Q14 AH Maths

| (13) | (a) Find an equation of the plane $\pi_1$ through the points $A(1, 1, 1)$, $B(2, -1, 1)$ and $C(0, 3, 3)$.  
(b) The plane $\pi_2$ has equation $x + 3y - z = 2$. Given that the point $(0, a, b)$ lies on both the planes $\pi_1$ and $\pi_2$, find the values of $a$ and $b$. Hence find an equation of the line of intersection of the planes $\pi_1$ and $\pi_2$.  
(c) Find the size of the acute angle between the planes $\pi_1$ and $\pi_2$. |

### Source: 2007 Q15 AH Maths

| (14) | Lines $L_1$ and $L_2$ are given by the parametric equations  
$L_1 : x = 2 + s, y = -s, z = 2 - s$  
$L_2 : x = -1 - 2t, y = t, z = 2 + 3t$.  
(a) Show that $L_1$ and $L_2$ do not intersect.  
(b) The line $L_3$ passes through the point $P(1, 1, 3)$ and its direction is perpendicular to the directions of both $L_1$ and $L_2$. Obtain parametric equations for $L_3$.  
(c) Find the coordinates of the point $Q$ where $L_3$ and $L_2$ intersect and verify that $P$ lies on $L_1$.  
(d) $PQ$ is the shortest distance between the lines $L_1$ and $L_2$. Calculate $PQ$. |