## Integration

**AH Maths Exam Questions**

Source: 2019 Specimen P1 Q4 AH Maths (Same as 2013 Q4b)

| (1) | The velocity, \( v \), of a particle \( P \) at time \( t \) is given by 
\[
   v = e^{3t} + 2e^t.
\]
Find the distance covered by \( P \) between \( t = 0 \) and \( t = \ln 3 \).

**Answer:** 
Distance covered = \( 12\frac{2}{3} \) units

Source: 2018 Q8 AH Maths

| (2) | Using the substitution \( u = \sin \theta \), or otherwise, evaluate 
\[
   \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\sin^4 \theta \cos \theta \, d\theta.
\]

**Answer:** \( \frac{31}{80} \) or 0.3875
On a suitable domain, a curve is defined by the equation $4x^2 + 9y^2 = 36$. A section of the curve in the first quadrant, illustrated in the diagram below, is rotated $360^\circ$ about the $y$-axis.

Calculate the exact value of the volume generated.

**Answer:** $Volume = 12\pi \text{ units}^3$

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Express $\frac{3x + 32}{(x + 4)(6 - x)}$ in partial fractions and hence evaluate $\int_3^4 \frac{3x + 32}{(x + 4)(6 - x)} \, dx$.

Give your answer in the form $\ln \left( \frac{\text{p}}{\text{q}} \right)$.

**Answer:** $\ln \left( \frac{486}{49} \right)$
### Source: 2015 Q17 AH Maths

(5) Find \[ \int \frac{2x^3 - x - 1}{(x - 3)(x^2 + 1)} \, dx, \quad x > 3. \]

Answer:
\[
2x + 5 \ln|x - 3| + \frac{1}{2} \ln(x^2 + 1) + k
\]

### Source: 2014 Q10 AH Maths

(6) A semi-circle with centre (1, 0) and radius 2, lies on the x-axis as shown. Find the volume of the solid of revolution formed when the shaded region is rotated completely about the x-axis.

Answer: \( \text{Volume} = 9\pi \text{ units}^3 \)
### Source: 2014 Q12 AH Maths

| (7) | Use the substitution \( x = \tan \theta \) to determine the exact value of 
\[
\int_{0}^{1} \frac{dx}{\left(1 + x^2\right)^{\frac{3}{2}}}. 
\]
| Answer: \( \frac{1}{\sqrt{2}} \) |

### Source: 2013 Q4 AH Maths

| (8) | The velocity, \( v \), of a particle \( P \) at time \( t \) is given by 
\[
v = e^{3t} + 2e^t.
\]
(a) Find the acceleration of \( P \) at time \( t \).
(b) Find the distance covered by \( P \) between \( t = 0 \) and \( t = \ln 3 \).
| Answers: (a) \( a = 3e^{3t} + 2e^t \)  (b) \( \frac{38}{3} \) or \( 12\frac{2}{3} \) |

### Source: 2013 Q6 AH Maths

| (9) | Integrate \( \frac{\sec^2 3x}{1 + \tan 3x} \) with respect to \( x \).
| Answer: \( \frac{1}{3} \ln |1 + \tan 3x| + c \) |
(10) Use the substitution $x = 4 \sin \theta$ to evaluate $\int_0^2 \sqrt{16 - x^2} \, dx$.

Answer: $\frac{4\pi}{3} + 2\sqrt{3} = 7.65$ (2 decimal places)

(11) A new board game has been invented and the symmetrical design on the board is made from four identical “petal” shapes. One of these petals is the region enclosed between the curves $y = x^2$ and $y^2 = 8x$ as shown shaded in diagram 1 below.

Calculate the area of the complete design, as shown in diagram 2.

The counter used in the game is formed by rotating the shaded area shown in diagram 1 above, through 360° about the $y$-axis. Find the volume of plastic required to make one counter.

Answer: $A = \frac{32}{3}$ units$^2$ $V = \frac{24\pi}{5}$ units$^3$

(12) Show that

$$\int_{\ln^2 2}^{\ln^2 3} \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx = \ln \frac{9}{5}.$$

Answer: Prove LHS = RHS
(13) Use the substitution $x = 2 \sin \theta$ to obtain the exact value of $\int_0^{\frac{\pi}{2}} \frac{x^2}{\sqrt{4-x^2}} \, dx$.
(Note that $\cos 2A = 1 - 2 \sin^2 A$.)

Answer: $\text{Exact Value} = \frac{\pi}{2} - 1$

(14) Express $\frac{12x^2 + 20}{x(x^2 + 5)}$ in partial fractions.
Hence evaluate $\int_1^2 \frac{12x^2 + 20}{x(x^2 + 5)} \, dx$.

Answers: $\frac{4}{x} + \frac{8x}{x^2 + 5}$ \quad $4\ln 3 \approx 4.39$

(15) Write down the derivative of $\tan x$.
Show that $1 + \tan^2 x = \sec^2 x$.
Hence obtain $\int \tan^2 x \, dx$.

Answers:

$$\frac{d}{dx} (\tan x) = \sec^2 x.$$ $1 + \tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \sec^2 x.$$

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$$
$$= \tan x - x + c$$
(16) A body moves along a straight line with velocity \( v = t^3 - 12t^2 + 32t \) at time \( t \).

(a) Obtain the value of its acceleration when \( t = 0 \).

(b) At time \( t = 0 \), the body is at the origin \( O \). Obtain a formula for the displacement of the body at time \( t \).

Show that the body returns to \( O \), and obtain the time, \( T \), when this happens.

Answers:

(a) Acceleration = 32 when \( t = 0 \)

(b) Displacement of body at time \( t \): \( x(t) = \frac{t^4}{4} - 4^3 + 16t^2 \)

The body returns to \( O \) when \( t = 8 \)

Source: 2007 Q10 AH Maths

(17) Use the substitution \( u = 1 + x^2 \) to obtain \( \int_0^1 \frac{x^3}{(1 + x^2)^4} \, dx \).

A solid is formed by rotating the curve \( y = \frac{x^{3/2}}{(1+x^2)^2} \) between \( x = 0 \) and \( x = 1 \) through 360° about the \( x \)-axis. Write down the volume of this solid.

Answers: \( \frac{1}{24}, \quad \text{Volume} = \frac{\pi}{24} \)