Further Differential Equations
AH Maths Exam Questions

Source: 2019 Specimen P2 Q12 AH Maths

Find the particular solution of the differential equation
\[ \frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = 6e^{2x} \]
given \( y = 4 \) and \( \frac{dy}{dx} = 7 \) when \( x = 0 \).

Answer:

<table>
<thead>
<tr>
<th>Generic scheme</th>
<th>Illustrative scheme</th>
<th>Max mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. solve auxiliary equation</td>
<td>( m = 2 ) twice</td>
<td>10</td>
</tr>
<tr>
<td>2. state complementary function</td>
<td>( y = Ae^{2x} + Bxe^{2x} )</td>
<td></td>
</tr>
<tr>
<td>3. state form of particular integral</td>
<td>( y = Cxe^{2x} )</td>
<td></td>
</tr>
<tr>
<td>4. find first derivative of particular integral</td>
<td>( \frac{dy}{dx} = 2Cxe^{2x} + 2Cxe^{2x} )</td>
<td></td>
</tr>
<tr>
<td>5. find second derivative</td>
<td>( \frac{d^2 y}{dx^2} = 4Cxe^{2x} + 8Cxe^{2x} + 2Ce^{2x} )</td>
<td></td>
</tr>
<tr>
<td>6. determine coefficient of particular integral</td>
<td>( C = 3 )</td>
<td></td>
</tr>
<tr>
<td>7. state general solution</td>
<td>( y = Ae^{2x} + Bxe^{2x} + 3xe^{2x} )</td>
<td></td>
</tr>
<tr>
<td>8. find derivative of general solution</td>
<td>( \frac{dy}{dx} = 2Ae^{2x} + Bxe^{2x} + 2Bxe^{2x} + 6xe^{2x} + 6xe^{2x} )</td>
<td></td>
</tr>
<tr>
<td>9. find one constant</td>
<td>( A = 4 ) or ( B = -1 )</td>
<td></td>
</tr>
<tr>
<td>10. find second constant and state particular solution</td>
<td>( y = 4e^{2x} - xe^{2x} + 3xe^{2x} )</td>
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</tbody>
</table>
Find the particular solution of the differential equation

\[
\frac{d^2 y}{dx^2} + 11 \frac{dy}{dx} + 28 y = 0
\]

given that \( y = 0 \) and \( \frac{dy}{dx} = 9 \), when \( x = 0 \).

**Answers:**

<table>
<thead>
<tr>
<th>Generic scheme</th>
<th>Illustrative scheme</th>
<th>Max mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 1 solve auxiliary equation</td>
<td>• 1 ( m = -4, -7 )</td>
<td>5</td>
</tr>
<tr>
<td>• 2 state general solution</td>
<td>• 2 ( y = Ae^{-4x} + Be^{-7x} )</td>
<td></td>
</tr>
<tr>
<td>• 3 differentiate</td>
<td>• 3 ( \frac{dy}{dx} = -4Ae^{-4x} - 7Be^{-7x} )</td>
<td></td>
</tr>
<tr>
<td>• 4 form equations and solve for a constant stated or implied at</td>
<td>• 4 ( A = 3 ) or ( B = -3 )</td>
<td></td>
</tr>
<tr>
<td>• 5 find second constant and state particular solution</td>
<td>• 5 ( y = 3e^{-4x} - 3e^{-7x} )</td>
<td></td>
</tr>
</tbody>
</table>
Source: 2018 Q15b AH Maths

(a) Use integration by parts to find \( \int x \sin 3x \, dx \).

(b) Hence find the particular solution of

\[
\frac{dy}{dx} - \frac{2}{x} y = x^3 \sin 3x, \quad x \neq 0
\]

given that \( x = \pi \) when \( y = 0 \).

Express your answer in the form \( y = f(x) \).

**Answers:**

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong> start integration by parts 1,2,3,4,6</td>
<td><strong>4</strong> ( e^{\int \frac{2}{x} , dx} )</td>
</tr>
<tr>
<td><strong>2</strong> complete integration by parts 1,2,3,4,6</td>
<td><strong>5</strong> ( \frac{1}{x^2} )</td>
</tr>
<tr>
<td><strong>3</strong> complete integration 1,2,3,4,5,6</td>
<td><strong>6</strong> ( \frac{d}{dx} \left( \frac{1}{x^2} y \right) = \frac{1}{x^2} (x^3 \sin 3x) ) stated or implied at <strong>4</strong></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{dy}{dx} & = x^3 \sin 3x \\
\int y \, dx & = \int x^3 \sin 3x \, dx \\
\end{align*}
\]

**7** \( \frac{1}{x} y = \int x \sin 3x \, dx \)

| **8** integrate 4,5,6 | **8** \( \frac{1}{x^2} y = -\frac{x}{3} \cos 3x + \frac{1}{9} \sin 3x + c \) |
| **9** evaluate constant 4,6,7,8 | **9** \( c = -\frac{\pi}{3} \) |
| **10** form particular solution 4,6,7,8 | **10** \( y = -\frac{x^3}{3} \cos 3x + \frac{x^2}{9} \sin 3x - \frac{\pi x^2}{3} \) |
Find the particular solution of the differential equation

\[ \frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9 y = 8 \sin x + 19 \cos x \]

given that \( y = 7 \) and \( \frac{dy}{dx} = \frac{1}{2} \) when \( x = 0 \).

**Answer:**

1. construct auxiliary equation
2. solve auxiliary equation and state CF
3. state PI
4. obtain first and second derivatives of PI
5. substitute
6. derive equations
7. obtain both constants of PI
8. differentiate general solution
9. determine first constant of general solution
10. determine second constant and state particular solution

\[
\begin{align*}
\text{Answer:} & \\
\text{1. } & m^2 - 6m + 9 = 0 \\
\text{2. } & y = Ae^{3x} + Bxe^{3x} \\
\text{3. } & y = C \sin x + D \cos x \\
\text{4. } & \frac{dy}{dx} = C \cos x - D \sin x \\
\text{5. } & -C \sin x - D \cos x - 6(C \cos x - D \sin x) + 9(C \sin x + D \cos x) = 8 \sin x + 19 \cos x \\
\text{6. } & 8C + 6D = 8 \\
\text{7. } & -6C + 8D = 19 \\
\text{8. } & C = -\frac{1}{2}, \quad D = 2 \\
\text{9. } & \frac{dy}{dx} = 3Ae^{3x} + Be^{3x} + 3Bxe^{3x} - \frac{1}{2} \cos x - 2 \sin x \\
\text{10. } & A = 5 \text{ or } B = -14 \\
\text{10. } & y = 5e^{3x} - 14xe^{3x} - \frac{1}{2} \sin x + 2 \cos x
\end{align*}
\]
(5) Solve the differential equation

\[ \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 6y = 12x^2 + 2x - 5 \]

given \( y = -6 \) and \( \frac{dy}{dx} = 3 \), when \( x = 0 \).

Answer:

| 1. state auxiliary equation | 1. \( m^2 + 5m + 6 = 0 \) |
| 2. solve auxiliary equation and state complementary function | 2. \( y = Ae^{-3x} + Be^{-2x} \) |
| 3. construct particular integral | 3. \( y = Cx^2 + Dx + E \) |
| 4. differentiate particular integral | 4. \( \frac{dy}{dx} = 2Cx + D \) and \( \frac{d^2 y}{dx^2} = 2C \) |
| 5. calculate one coefficient of the particular integral | 5. \( C = 2 \) |
| 6. calculate remaining coefficients | 6. \( D = -3, E = 1 \) |
| 7. differentiate general solution | \( y = Ae^{-3x} + Be^{-2x} + 2x^2 - 3x + 1 \) |
| 8. construct equations using given conditions | \( \frac{dy}{dx} = -3Ae^{-3x} - 2Be^{-2x} + 4x - 3 \) |
| 9. Find one coefficient | \( A + B = -7 \) and \( 3A + 2B = -6 \) or equivalent |
| 10. Find other coefficient and state particular solution | \( A = 8 \) or \( B = -15 \) |
|  | \( y = 8e^{-3x} - 15e^{-2x} + 2x^2 - 3x + 1 \) |
Solve the second order differential equation
\[ \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 10y = 3e^{2x} \]
given that when \( x = 0 \), \( y = 1 \) and \( \frac{dy}{dx} = 0 \).

Answer:

\[
\begin{align*}
\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 10y &= 3e^{2x} \\
\quad m^2 + 2m + 10 &= 0 \\
\quad m = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 10}}{2} &= -1 \pm 3i \\

y &= e^{-x} (A \cos 3x + B \sin 3x) \quad \text{OR} \quad y = ae^{(-1+3i)x} + Be^{(-1-3i)x} \\
\text{try} \quad y &= Ce^{2x} \\
\frac{dy}{dx} &= 2Ce^{2x} \\
\frac{d^2y}{dx^2} &= 4Ce^{2x} \\
4Ce^{2x} + 4Ce^{2x} + 10Ce^{2x} &= 3e^{2x} \\
C &= \frac{1}{6} \\
y &= Ae^{-x} \cos 3x + Be^{-x} \sin 3x + \frac{1}{6}e^{2x} \\
1 &= A + \frac{1}{6} , \quad A = \frac{5}{6} \\
\frac{dy}{dx} &= -Ae^{-x} \cos 3x - 3Ae^{-x} \sin 3x - Be^{-x} \sin 3x + 3Be^{-x} \cos 3x + \frac{1}{3}e^{2x} \\
0 &= -(A) + 3B + \frac{1}{3} , \quad \frac{5}{6} - \frac{1}{3} = 3B , \quad B = \frac{1}{6} \\

\text{So particular solution is: } y = \frac{5}{6}e^{-x} \cos 3x + \frac{1}{6}e^{-x} \sin 3x + \frac{1}{6}e^{2x}
\end{align*}
\]

1. correct auxiliary equation.
2. solves correctly\(^1\).
3. appropriate complementary function.
4. particular integral
5. for \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \)
6. for finding \( C \)
7. combine CF and PI for general solution\(^3\).
8. value of \( A \).
9. for differentiating correctly\(^3\).
10. value of \( B \) and statement of final answer\(^4\).
Find the solution $y = f(x)$ to the differential equation

$$4\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} + y = 0$$

given that $y = 4$ and $\frac{dy}{dx} = 3$ when $x = 0$.

Answer:

<table>
<thead>
<tr>
<th>Expected Answer/s</th>
<th>Max Mark</th>
<th>Additional Guidance</th>
</tr>
</thead>
</table>
| $4m^2 - 4m + 1 = 0$ | 6 | 1 Correct auxiliary equation.
| $(2m - 1)^2 = 0$ | | 2 Correct solution of auxiliary equation.
| $m = \frac{1}{2}$ | | 3 Statement of general solution/complementary function.
| $y = Ae^{\frac{1}{2}x} + Bxe^{\frac{1}{2}x}$ | | 4 Correct evaluation of $A$. \text{4,4,5,6}
| $y = 4$ when $x = 0$ gives $4 = A.1 + 0$, so $A = 4$ | | 5 Correct differentiation of G.S.
| $\frac{dy}{dx} = \frac{1}{2}Ae^{\frac{1}{2}x} + Be^{\frac{1}{2}x} + \frac{1}{2}Bxe^{\frac{1}{2}x}$ | | 6 Substitution to obtain $B$ and particular solution.
| $\frac{dy}{dx} = 3$ when $x = 0$ gives | |  |
Solve the differential equation
\[
\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 4e^{3x}, \text{ given that } y = 1 \text{ and } \frac{dy}{dx} = -1 \text{ when } x = 0.
\]

**Answer:**

\[
\begin{align*}
m^2 - 6m + 9 &= 0 \\
(m - 3)^2 &= 0 \\
m &= 3
\end{align*}
\]

C.F. \[y = Ae^{3x} + Bxe^{3x}\]

P.I. Try \[y = Cx^2e^{3x}\]

\[
\begin{align*}
\frac{dy}{dx} &= 2Cxe^{3x} + 3Cx^2e^{3x} \\
\frac{d^2 y}{dx^2} &= 2Ce^{3x} + 6Cxe^{3x} + 6Cxe^{3x} + 9Cx^2e^{3x} \\
2Ce^{3x} + 6Cxe^{3x} + 6Cxe^{3x} + 9Cx^2e^{3x} - 6(2Cxe^{3x} + 3Cx^2e^{3x}) + 9Cx^2e^{3x} &= 4e^{3x} \\
2Ce^{3x} &= 4e^{3x} \Rightarrow C = 2
\end{align*}
\]

G.S. \[y = Ae^{3x} + Bxe^{3x} + 2x^2e^{3x}\]

\[
\begin{align*}
\frac{dy}{dx} &= 3Ae^{3x} + Be^{3x} + 3Bxe^{3x} + 4xe^{3x} + 6x^2e^{3x} \\
\text{When } x = 0, y = 1 & \quad A = 1 \\
\frac{dy}{dx} &= -1 & \quad -1 = 3 + B \Rightarrow B = -4 \\
P.S. & \quad y = e^{3x} - 4xe^{3x} + 2x^2e^{3x}
\end{align*}
\]

\[\text{• Correct auxiliary equation (or equivalent).}^{11}\]

\[\text{• Correct solution of auxiliary equation and statement of complimentary function.}^{2}\]

\[\text{• Correct form of particular integral.}^{1,7}\]

\[\text{• Correct first derivative of P.I.}^{2,3}\]

\[\text{• Correct differentiation of first derivative.}^{3}\]

\[\text{• For correctly substituting expressions for both derivatives.}^{6}\]

\[\text{• For correctly solving to obtain C.}^{5}\]

\[\text{• Correct collation of above answers to obtain full General Solution.}^{6}\]

\[\text{• Derivative of G.S.}^{9}\]

\[\text{• Use of i.c.s to find first constant correctly.}^{10}\]

\[\text{• Second constant.}^{11}\]

States solution.\[6\]
(9)  

(a) Express \( \frac{1}{(x-1)(x+2)^2} \) in partial fractions.

(b) Obtain the general solution of the differential equation

\[ (x-1) \frac{dy}{dx} - y = \frac{x-1}{(x+2)^2}, \]

expressing your answer in the form \( y = f(x) \).

**Answers:**

(a) \[ \frac{1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} \]

\[ 1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1) \]

\[ x = 1 \Rightarrow A = \frac{1}{9} \]

\[ x = -2 \Rightarrow C = -\frac{1}{3} \]

\[ x = 0 \Rightarrow 1 = \frac{1}{9} - 2B + \frac{1}{3} \Rightarrow B = -\frac{1}{9} \]

\[ \therefore \frac{1}{(x-1)(x+2)^2} = \frac{1}{9} \left( \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2} \right) \]

(b) \[ (x-1) \frac{dy}{dx} - y = \frac{x-1}{(x+2)^2} \]

\[ \frac{dy}{dx} - \frac{1}{x-1}y = \frac{1}{(x+2)^2} \]

for rearranging

Integrating factor: \( \exp \left( \int -\frac{1}{x-1} \, dx \right) \)

\[ = \exp \left( -\ln |x - 1| \right) = (x - 1)^{-1} \]

\[ \frac{1}{(x-1)} \frac{dy}{dx} - \frac{1}{(x-1)^2}y = \frac{1}{(x-1)(x+2)^2} \]

\[ \frac{dy}{dx} \left( \frac{y}{x-1} \right) = \frac{1}{(x-1)(x+2)^2} \]

\[ = \frac{1}{9} \left( \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2} \right) \]

\[ \frac{y}{x-1} = \frac{1}{9} \left( \ln |x - 1| - \ln |x + 2| + \frac{3}{x + 2} \right) + c \]

\[ y = \frac{x-1}{9} \left( \ln |x - 1| - \ln |x + 2| + \frac{3}{x + 2} \right) + c(x - 1) \]

\[ = \frac{x-1}{9} \left( \ln \left| \frac{x-1}{x+2} \right| + \frac{3}{x+2} \right) + c(x - 1) \]
Find the general solution of the differential equation
\[
\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12.
\]

Find the particular solution for which \( y = -\frac{3}{2} \) and \( \frac{dy}{dx} = \frac{1}{2} \) when \( x = 0 \).

**Answer:**

| Auxiliary equation |  
|--------------------|---|
| \( m^2 - m - 2 = 0 \) | 1 |
| \((m - 2)(m + 1) = 0 \) |  |
| \( m = -1 \) or \( 2 \) | 1 |

Complementary function is: \( y = Ae^{-x} + Be^{2x} \)

The particular integral has the form \( y = Ce^x + D \)

\[
y = Ce^x + D \Rightarrow \frac{dy}{dx} = Ce^x \\
\Rightarrow \frac{d^2y}{dx^2} = Ce^x
\]

Hence we need:

\[
\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12 \\
[Ce^x] - [Ce^x] - 2[Ce^x + D] = e^x + 12 \\
-2Ce^x - 2D = e^x + 12
\]

Hence \( C = -\frac{1}{2} \) and \( D = -6 \).

So the General Solution is

\[
y = Ae^{-x} + Be^{2x} - \frac{1}{2}e^x - 6.
\]

\[
x = 0 \text{ and } y = -\frac{1}{2} \Rightarrow \quad A + B - \frac{1}{2} - 6 = -\frac{3}{2}
\]

\[
x = 0 \text{ and } \frac{dy}{dx} = \frac{1}{2} \Rightarrow \quad -A + 2B - \frac{1}{2} = \frac{1}{2}
\]

\[
3B - 7 = -1 \Rightarrow B = 2 \Rightarrow A = 3
\]

So the particular solution is

\[
y = 3e^{-x} + 2e^{2x} - \frac{1}{2}e^x - 6.
\]
Obtain the general solution of the equation

\[
\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 5y = 0.
\]

Hence obtain the solution for which \( y = 3 \) when \( x = 0 \) and \( y = e^{-\pi} \) when \( x = \frac{\pi}{2} \).

**Answer:**

\[
\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 5y = 0
\]

\[
m^2 + 4m + 5 = 0
\]

\[
(m + 2)^2 = -1
\]

\[
m = -2 \pm i
\]

1

The general solution is

\[
y = e^{-2x}(A \cos x + B \sin x)
\]

1M

appropriate CF for accuracy

\[
x = 0, y = 3 \implies 3 = A
\]

1

\[
x = \frac{\pi}{2}, y = e^{-\pi} \implies e^{-\pi} = e^{-\pi}(3 \cos \frac{\pi}{2} + B \sin \frac{\pi}{2})
\]

\[
\implies B = 1
\]

1

The particular solution is:

\[
y = e^{-2x}(3 \cos x + \sin x).
\]

1
(a) Solve the differential equation

\[(x + 1) \frac{dy}{dx} - 3y = (x + 1)^4\]

given that \(y = 16\) when \(x = 1\), expressing the answer in the form \(y = f(x)\).

(b) Hence find the area enclosed by the graphs of \(y = f(x)\), \(y = (1 - x)^4\) and the \(x\)-axis.

Answers:

(a) \[(x + 1) \frac{dy}{dx} - 3y = (x + 1)^4\]

\[\frac{dy}{dx} - \frac{3}{x + 1}y = (x + 1)^3\]

Integrating factor:

since \[\int \frac{-3}{x + 1} dx = -3 \ln(x + 1)\].

Hence the integrating factor is \((x + 1)^{-3}\).

\[\frac{1}{(x + 1)^3} \frac{dy}{dx} - \frac{3}{(x + 1)^4} y = 1\]

\[\frac{d}{dx} \left( \frac{1}{x + 1} y \right) = 1\]

\[\frac{y}{(x + 1)^3} = \int 1 \, dx\]

\[= x + C\]

\(y = 16\) when \(x = 1\), so \(2 = 1 + C \Rightarrow C = 1\). Hence

\[y = (x + 1)^4\]

(b) \[(x + 1)^4 = (1 - x)^4\]

\[x + 1 = 1 - x \Rightarrow x = 0\]

or \(x + 1 = -1 + x\) which has no solutions.

\[\text{Area} = \int_{-1}^{1} (x + 1)^4 \, dx + \int_{0}^{1} (1 - x)^4 \, dx\]

\[= 2 \int_{-1}^{0} (x + 1)^4 \, dx\]

\[= \frac{2}{5} \left[ (x + 1)^5 \right]_{-1}^{0} = \frac{2}{5} - 0 = \frac{2}{5}\]
Obtain the general solution of the differential equation
\[ \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 2x^2. \]

Given that \( y = \frac{1}{2} \) and \( \frac{dy}{dx} = 1 \), when \( x = 0 \), find the particular solution.

Answer:

\[ \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 2x^2 \]

\( m^2 - 3m + 2 = 0 \)

\( (m - 1)(m - 2) = 0 \)

\( m = 1 \) or \( m = 2 \)

Complementary function: \( y = Ae^x + Be^{2x} \)

For particular integral try \( y = ax^2 + bx + c \)

\[ \Rightarrow \frac{dy}{dx} = 2ax + b; \quad \frac{d^2y}{dx^2} = 2a \]

Hence require

\[ 2a - 3(2ax + b) + 2(ax^2 + bx + c) = 2x^2 \]

\[ 2ax^2 + (-6a + 2b)x + (2a - 3b + 2c) = 2x^2 \]

\[ \Rightarrow a = 1; \quad b = 3; \quad c = \frac{7}{2} \]

General solution is: \( y = Ae^x + Be^{2x} + x^2 + 3x + \frac{7}{2} \)

When \( x = 0 \), \( y = \frac{1}{2} \) and \( \frac{dy}{dx} = 1 \).

\[ \frac{1}{2} = A + B + \frac{7}{2} \Rightarrow A + B = -3 \]

\[ \frac{dy}{dx} = Ae^x + 2Be^{2x} + 2x + 3 \Rightarrow 1 = A + 2B + 3 \Rightarrow A + 2B = -2 \]

\[ B = 1 \quad A = -4 \]

Particular solution is

\[ y = -4e^x + e^{2x} + x^2 + 3x + \frac{7}{2}. \]
Obtain the general solution of the equation \( \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + 9y = e^{2x} \).

Answer:

\[
\frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + 9y = e^{2x}
\]

Auxiliary equation: \( m^2 + 6m + 9 = 0 \)

\[ \text{So } (m + 3)^2 = 0 \text{ giving } m = -3. \]

Complementary function:

\[ y = (A + Bx)e^{-3x} \]

For the Particular Integral try \( y = ke^{2x} \)

\[ \Rightarrow \frac{dy}{dx} = 2ke^{2x}; \frac{d^2y}{dx^2} = 4ke^{2x} \]

\[ 4ke^{2x} + 12ke^{2x} + 9ke^{2x} = e^{2x} \Rightarrow 25k = 1 \]

Hence the General Solution is:

\[ y = (A + Bx)e^{-3x} + \frac{1}{25}e^{2x} \]
<table>
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<th>Source: 2006 Q8 AH Maths</th>
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<tbody>
<tr>
<td>Solve the differential equation</td>
</tr>
</tbody>
</table>
| \[
\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0
\] |
| given that when \( x = 0 \), \( y = 0 \) and \( \frac{dy}{dx} = 2 \). |
| **Answer:** |
| \[
\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0
\]
| A.E. \( m^2 + 2m + 2 = 0 \) | 1 |
| \( m = -2 \pm \sqrt{4 - 8} \) = \(-1 \pm i\) | 1 |
| General solution is |
| \( y = e^{-x}(A \cos x + B \sin x) \) | 1 |
| \( y = 0 \) when \( x = 0 \) \( \Rightarrow \) 0 = \( A \) | 1 |
| \( \frac{dy}{dx} = -e^{-x}B \sin x + e^{-x}B \cos x \) | 1 |
| 2 = 0 + \( B \) | 1 |
| The solution is \( y = 2e^{-x} \sin x \). |
Obtain the general solution of the differential equation \( \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 20 \sin x \)

Hence find the particular solution for which \( y = 0 \) and \( \frac{dy}{dx} = 0 \) when \( x = 0 \).

**Answer:**

Let \( y = e^{mx} \), then the auxiliary equations is

\[
m^2 - 3m + 2 = 0
\]
\[
(m - 1)(m - 2) = 0
\]

\( m = 1 \) or \( m = 2 \)

The Complementary Function is \( y = Ae^x + Be^{2x} \).

For the Particular Integral, try \( y = a \sin x + b \cos x \).

\[
\frac{dy}{dx} = a \cos x - b \sin x
\]
\[
\frac{d^2y}{dx^2} = -a \sin x - b \cos x
\]

Substituting:

\[
(-a \sin x - b \cos x) - 3(a \cos x - b \sin x) + 2(a \sin x + b \cos x) = 20 \sin x
\]
\[
(-a + 3b + 2a) \sin x + (-b - 3a + 2b) \cos x = 20 \sin x
\]
\[
a + 3b = 20; \quad -3a + b = 0
\]

\( a = 2; \quad b = 6 \).

The general solution is

\[y = Ae^x + Be^{2x} + 2 \sin x + 6 \cos x\]

\[
\frac{dy}{dx} = Ae^x + 2Be^{2x} + 2 \cos x - 6 \sin x
\]

\( y = 0 \) when \( x = 0 \) so \( A + B + 6 = 0 \).

\( \frac{dy}{dx} = 0 \) when \( x = 0 \) so \( A + 2B + 2 = 0 \).

\( B = 4; \quad A = -10 \)

The particular solution is

\[y = -10e^x + 4e^{2x} + 2 \sin x + 6 \cos x.\]