## Functions & Graphs

### AH Maths Exam Questions

<table>
<thead>
<tr>
<th>Source: 2019 Specimen P1 Q8 AH Maths</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(1)</strong> A function is defined on a suitable domain by ( f(x) = \frac{3x^2 + 2}{x^2 - 2} ).</td>
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<td><strong>(a)</strong> Obtain equations for the asymptotes of the graph of ( y = f(x) ).</td>
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<td><strong>(b)</strong> Determine whether the graph of ( y = f(x) ) has any points of inflection. Justify your answer.</td>
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</tbody>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>(2)</strong> The function ( f'(x) ) is defined by ( f(x) = x^2 - a^2 ). The graph of ( y = f'(x) ) is shown in the diagram.</td>
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</tbody>
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![Diagram of a parabola with vertex at the origin and intersecting the x-axis at -a and a.]

| **(a)** State whether \( f(x) \) is odd, even or neither. Give a reason for your answer. |
| **(b)** Sketch the graph of \( y = |f(x)| \). |
In the diagram below part of the graph of \( y = f(x) \) has been omitted.

The point \((-1, -2)\) lies on the graph and the line \( y = \frac{1}{2}x - 3 \) is an asymptote.

(a) Copy and complete the diagram, including any asymptotes and any points you know to be on the graph.

(b) \( g(x) = |f(x)| \). On a separate diagram, sketch \( g(x) \).
   Include known asymptotes and points.

(c) State the range of values of \( f'(x) \) given that \( f'(0) = 2 \).
Below is a diagram showing the graph of a linear function, \( y = f(x) \).

\[ y \]
\[ O \]
\[ x \]
\[ -c \]

On separate diagrams show:

(a) \( y = |f(x) - c| \)

(b) \( y = |2f(x)| \)

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For some function, \( f \), define
\[
    g(x) = f(x) + f(-x) \quad \text{and} \quad h(x) = f(x) - f(-x).
\]

Show that \( g(x) \) is an even function and that \( h(x) \) is an odd function.
Hence show that \( f(x) \) can be expressed as the sum of an even and an odd function.
The function $f(x)$ is defined for all $x \geq 0$.
The graph of $y = f(x)$ intersects the $y$-axis at $(0, c)$, where $0 < c < 5$.
The graph of the function and its asymptote, $y = x - 5$, are shown below.

(a) Copy the above diagram.
   On the same diagram, sketch the graph of $y = f^{-1}(x)$.
   Clearly show any points of intersection and any asymptotes.

(b) What is the equation of the asymptote of the graph of $y = f(x + 2)$?

(c) Why does your diagram show that the equation $x = f(f(x))$ has at least one solution?
Part of the straight line graph of a function $f(x)$ is shown.

(a) Sketch the graph of $f^{-1}(x)$, showing points of intersection with the axes.
(b) State the value of $k$ for which $f(x) + k$ is an odd function.
(c) Find the value of $h$ for which $|f(x + h)|$ is an even function.

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A function is defined by $f(x) = |x + 2|$ for all $x$.

(a) Sketch the graph of the function for $-3 \leq x \leq 3$.
(b) On a separate diagram, sketch the graph of $f'(x)$. 
The diagram shows part of the graph of a function $f(x)$. Sketch the graph of $|f^{-1}(x)|$ showing the points of intersection with the axes.

The diagram below shows part of the graph of a function $f(x)$. State whether $f(x)$ is odd, even or neither. Fully justify your answer.
The function \( f(x) \) is defined by
\[
f(x) = \frac{x^2 + 2x}{x^2 - 1} \quad (x \neq \pm 1).
\]

Obtain equations for the asymptotes of the graph of \( f(x) \).
Show that \( f(x) \) is a strictly decreasing function.
Find the coordinates of the points where the graph of \( f(x) \) crosses
(i) the \( x \)-axis and
(ii) the horizontal asymptote.
Sketch the graph of \( f(x) \), showing clearly all relevant features.

Part of the graph \( y = f(x) \) is shown below, where the dotted lines indicate asymptotes. Sketch the graph \( y = -f(x + 1) \) showing its asymptotes. Write down the equations of the asymptotes.
(a) The diagram shows part of the graph of \( f(x) = \tan^{-1} 2x \) and its asymptotes. State the equations of these asymptotes.

(b) Use integration by parts to find the area between \( f(x) \), the \( x \)-axis and the lines \( x = 0, x = \frac{1}{2} \).

(c) Sketch the graph of \( y = |f(x)| \) and calculate the area between this graph, the \( x \)-axis and the lines \( x = -\frac{1}{2}, x = \frac{1}{2} \).