Differentiation

AH Maths Exam Questions

Source: 2019 Specimen P1 Q2 AH Maths

(1) Given \( f(x) = 2x \tan x \), where \( 0 < x < \frac{\pi}{2} \), obtain \( f'' \left( \frac{\pi}{4} \right) \).

Source: 2019 Q1a,b AH Maths

(2) (a) Differentiate \( f(x) = x^6 \cot 5x \).

(b) Given \( y = \frac{2x^3 + 1}{x^3 - 4} \), find \( \frac{dy}{dx} \). Simplify your answer.

(c) For \( f(x) = \cos^{-1} 2x \) evaluate \( f' \left( \frac{\sqrt{3}}{4} \right) \).

Source: 2019 Q6 AH Maths

(3) A spherical balloon of radius \( r \) cm, \( r > 0 \), deflects at a constant rate of \( 60 \text{ cm}^3 \text{s}^{-1} \). Calculate the rate of change of the radius with respect to time when \( r = 3 \).

\[
\text{The volume of a sphere is given by } V = \frac{4}{3} \pi r^3.
\]
(4)  
(a) Given \( f(x) = \sin^{-1} 3x \), find \( f'(x) \).
(b) Differentiate \( y = \frac{e^{5x}}{7x + 1} \).
(c) For \( y \cos x + y^2 = 6x \), use implicit differentiation to find \( \frac{dy}{dx} \).

(5)  
On a suitable domain, a function is defined by \( f(x) = \frac{e^{x^2-1}}{x^2-1} \).

Find \( f''(x) \), simplifying your answer.

(6)  
(a) Differentiate \( y = x \tan^{-1} 2x \).
(b) Given \( f(x) = \frac{1-x^2}{1+4x^2} \), find \( f'(x) \), simplifying your answer.
(c) A curve is given by the parametric equations \( x = 6t \) and \( y = 1 - \cos t \).

Find \( \frac{dy}{dx} \) in terms of \( t \).
### Source: 2015 Q2 AH Maths

<table>
<thead>
<tr>
<th>(7)</th>
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| (a) For $y = \frac{5x + 1}{x^2 + 2}$, find $\frac{dy}{dx}$. Express your answer as a single, simplified fraction.  

(b) Given $f(x) = e^{2x}\sin^2 3x$, obtain $f'(x)$. |

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### Source: 2014 Q1 AH Maths

<table>
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| (a) Given 

$$f(x) = \frac{x^2 - 1}{x^2 + 1},$$

obtain $f'(x)$ and simplify your answer.  

(b) Differentiate $y = \tan^{-1}(3x^2)$. |

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### Source: 2014 Q13 AH Maths

<table>
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| The fuel efficiency, $F$, in km per litre, of a vehicle varies with its speed, $s$ km per hour, and for a particular vehicle the relationship is thought to be 

$$F = 15 + e^x(\sin x - \cos x - \sqrt{2}),$$  

where $x = \frac{\pi(s - 40)}{80}$,  

for speeds in the range $40 \leq s \leq 120$ km per hour.  

What is the greatest and least efficiency over the range and at what speeds do they occur? |
**Source: 2013 Q2 AH Maths**

(10) Differentiate \( f(x) = e^{\cos x} \sin^2 x \).  

**Source: 2012 Q1 AH Maths**

(11)  

(a) Given \( f(x) = \frac{3x+1}{x^2+1} \), obtain \( f'(x) \).

(b) Let \( g(x) = \cos^2 x \exp (\tan x) \). Obtain an expression for \( g'(x) \) and simplify your answer.

**Source: 2011 Q7 AH Maths**

(12) A curve is defined by the equation \( y = \frac{e^{\sin x} (2 + x)^3}{\sqrt{1-x}} \) for \( x < 1 \). Calculate the gradient of the curve when \( x = 0 \).

**Source: 2010 Q1 AH Maths**

(13) Differentiate the following functions.

(a) \( f(x) = e^x \sin x^2 \).

(b) \( g(x) = \frac{x^3}{1 + \tan x} \).
### Source: 2009 Q1a AH Maths

(14) Given $f(x) = (x + 1)(x - 2)^3$, obtain the values of $x$ for which $f'(x) = 0$.

(b) Calculate the gradient of the curve defined by $\frac{x^2}{y} + x = y - 5$ at the point $(3, -1)$.

### Source: 2008 Q10 AH Maths

(15) A body moves along a straight line with velocity $v = t^3 - 12t^2 + 32t$ at time $t$.

(a) Obtain the value of its acceleration when $t = 0$.

(b) At time $t = 0$, the body is at the origin $O$. Obtain a formula for the displacement of the body at time $t$.

Show that the body returns to $O$, and obtain the time, $T$, when this happens.

### Source: 2008 Q15 AH Maths

(16) Let $f(x) = \frac{x}{\ln x}$ for $x > 1$.

(a) Derive expressions for $f''(x)$ and $f'''(x)$, simplifying your answers.

(b) Obtain the coordinates and nature of the stationary point of the curve $y = f(x)$.

(c) Obtain the coordinates of the point of inflexion.

### Source: 2007 Q2 AH Maths

(17) Obtain the derivative of each of the following functions:

(a) $f(x) = \exp (\sin 2x)$;

(b) $y = 4^{(x^2 + 1)}$. 